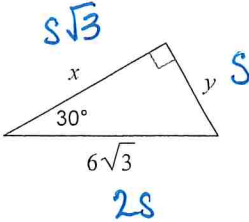
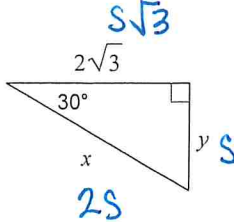
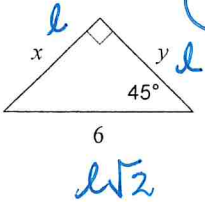


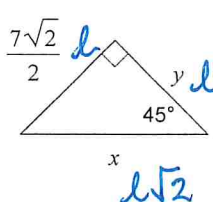
Focus - Section 5.8 Special Right Triangles

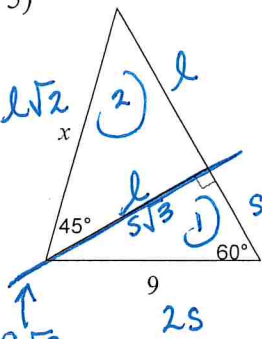
Find the missing side lengths. Leave your answers as radicals in simplest form.

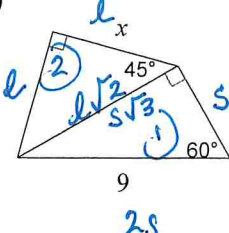
1)   $\frac{2s}{2} = \frac{6\sqrt{3}}{2}$   
 $s = 3\sqrt{3}$   
 $y = 3\sqrt{3}$   
 $x = 3\sqrt{3} \cdot \sqrt{3} = 9$

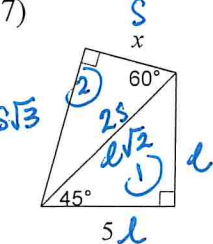
2)   $2\sqrt{3} = s\sqrt{3}$   
 $s = 2$   
 $x = 2(2) = 4$   
 $y = 2$

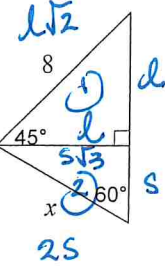
3)   $\frac{b}{\sqrt{2}} = \frac{l\sqrt{2}}{\sqrt{2}}$   
 $l = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 3\sqrt{2}$   
 $x = 3\sqrt{2}$   
 $y = 3\sqrt{2}$

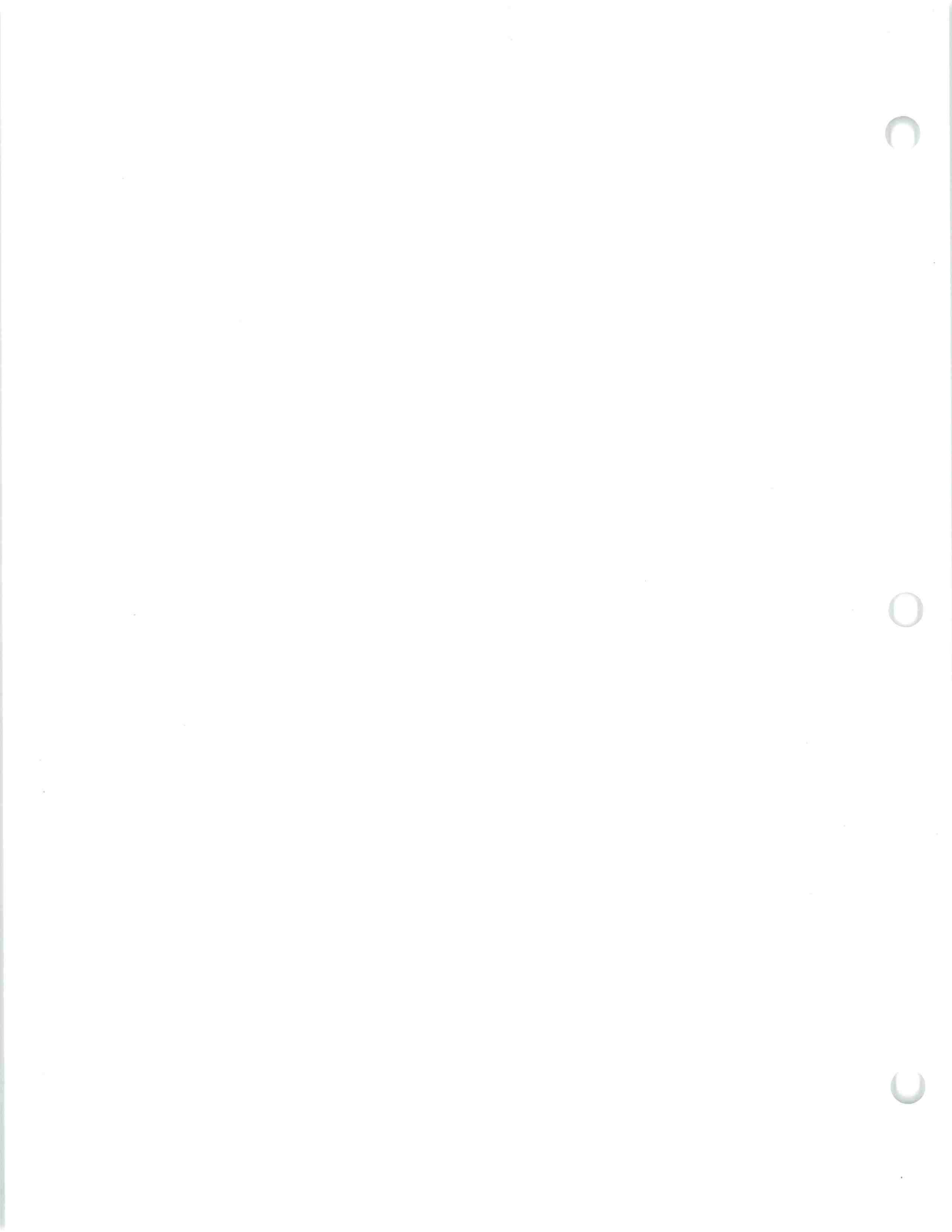
4)   $l = \frac{7\sqrt{2}}{2}$   
 $x = \frac{7\sqrt{2}}{2} \cdot \sqrt{2} = 7$   
 $y = \frac{7\sqrt{2}}{2}$

5)   $2s = 9$   
 $s = \frac{9}{2}$   
 $l = \frac{9\sqrt{3}}{2}$   
 $x = \frac{9\sqrt{3}}{2} \cdot \sqrt{2} = \frac{9\sqrt{6}}{2}$

6)   $2s = 9$   
 $s = \frac{9}{2}$   
 $l\sqrt{2} = \frac{9\sqrt{3}}{2}$   
 $l = \frac{9\sqrt{3}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{9\sqrt{6}}{4} = x$

7)   $l = 5$   
 $\frac{2s}{2} = \frac{5\sqrt{2}}{2}$   
 $s = \frac{5\sqrt{2}}{2}$   
 $x = \frac{5\sqrt{2}}{2}$

8)   $8 = l\sqrt{2}$   
 $l = \frac{8}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$   
 $\frac{4\sqrt{2}}{\sqrt{3}} = \frac{s\sqrt{3}}{\sqrt{3}}$   
 $s = \frac{4\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{6}}{3}$   
 $x = 2 \cdot \frac{4\sqrt{6}}{3} = \frac{8\sqrt{6}}{3}$



1) Given vertices A(-3, 7) B(4, 7) C(-3, -1)

Find the perimeter and area.

$P = (15 + \sqrt{113}) \text{ un.}$   
 $A = 28 \text{ un}^2$

$8^2 + 7^2 = x^2$

$64 + 49 = x^2$

$113 = x^2$

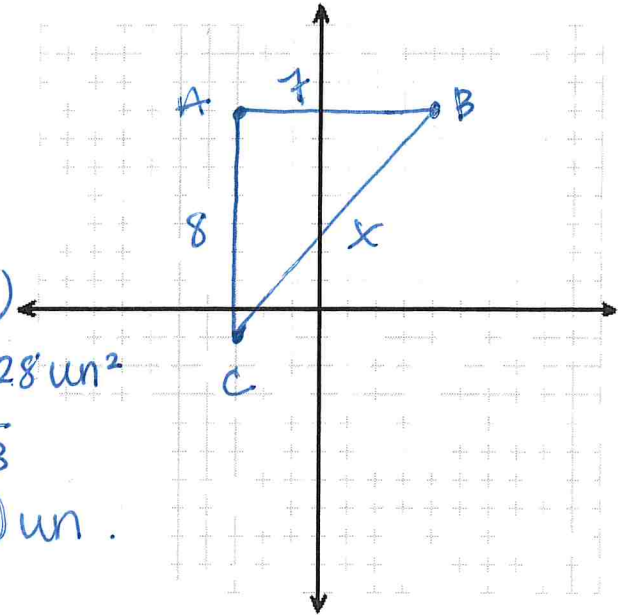
$x = \sqrt{113}$

$A = \frac{1}{2}(8 \times 7)$

$A = \frac{1}{2}(56) = 28 \text{ un}^2$

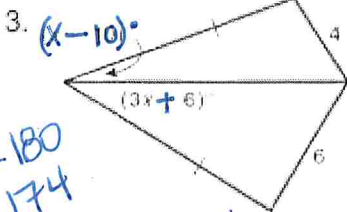
$P = 8 + 7 + \sqrt{113}$

$P = (15 + \sqrt{113}) \text{ un.}$



**★ Warm up ★**

Find the range of values for x.

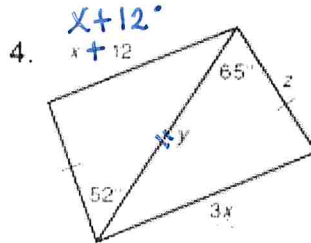


1)  $x - 10 > 0$   
 $x > 10$

2)  $3x + 6 > 0$   
 $3x > -6$

Hinge Thm  
3)  $x - 10 < 3x + 6$   
 $-10 < 2x + 6$   
 $-16 < 2x$   
 $-8 < x$

$10 < x < 58$



2)  $x + 12 > 0$   
 $x > -12$

1)  $x + 12 < 3x$  Hinge Thm

$12 < 2x$   
 $6 < x$   
 $x > 6$

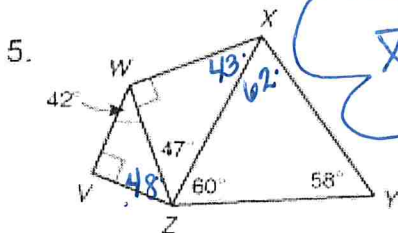
3)  $3x > 0$   
 $x > 0$

Triangle Inequality Thm

4)  $y + z > 3x$

$6 < x < \frac{1}{3}(y+z)$   $x < \frac{1}{3}(y+z)$

List the segments in order from smallest to largest in #5.



$\overline{VZ}, \overline{VW}, \overline{WZ}, \overline{WX},$   
 $\overline{XZ}, \overline{XY}, \overline{YZ}$

#6 Tell whether the following values would form a triangle:

$8 + 16 > 14$   
 $14 + 16 > 8$  ✓  
 $8 + 14 > 16$

14, 16, 8  
 $3x + 2, x^2, 2x$  when  $x = 4$  yes

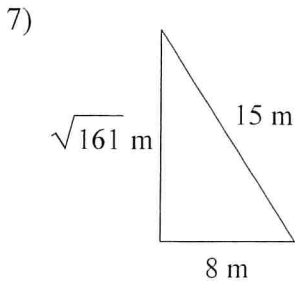
$3x + 2, x^2, 2x$  when  $x = 6$  no

20, 36, 12  
 $20 + 36 > 12$   
 $12 + 36 > 20$   
 $12 + 20 > 36$  ✗

State if the three numbers can be the measures of the sides of a triangle.

- 1) 20, 6, 12  $6+12 \not> 20$  no  
 2) 12, 11, 7 yes  
 3) 18, 7, 12 yes  
 4) 3, 9, 6 no  $3+6 \not> 9$   
 5) 10, 6, 3  $3+6 \not> 10$  no  
 6) 10, 11, 20 yes

State if each triangle is acute, obtuse, or right.

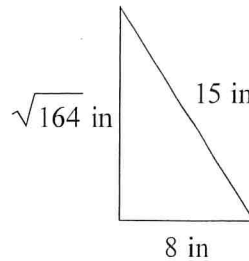


$$8^2 + (\sqrt{161})^2 \stackrel{?}{=} 15^2$$

$$64 + 161 = 225$$

$$225 = 225$$

Right

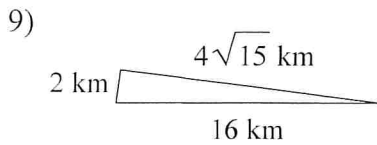


$$(\sqrt{164})^2 + 8^2 \stackrel{?}{=} 15^2$$

$$164 + 64 = 228$$

$$228 > 225$$

Acute



$$(2)^2 + (4\sqrt{15})^2 \stackrel{?}{=} 16^2$$

$$4 + (16)(15) = 244$$

$$244 < 256$$

Obtuse

$$\begin{array}{r} 3 \\ \times 16 \\ \hline 48 \\ 480 \\ \hline 240 \end{array}$$

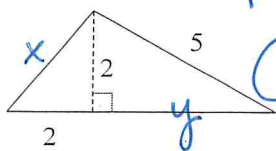
Two sides of a triangle have the following measures. Find the range of possible measures for the third side.

- 10) 11, 7, X  $4 < X < 18$
- $$\begin{array}{ll} 11 + X > 7 & X > \cancel{4} \\ 11 + 7 > X & X < 18 \\ X + 7 > 11 & X > 4 \end{array}$$
- 11) 12, 12, X  $0 < X < 24$
- $$\begin{array}{ll} 12 + 12 > X & X < 24 \\ 12 + X > 12 & X > 0 \end{array}$$
- 12) 6, 9, X  $3 < X < 15$
- $$\begin{array}{ll} 6 + 9 > X & 15 > X \\ 6 + X > 9 & X > 3 \\ 9 + X > 6 & X > \cancel{3} \end{array}$$
- 13) 10, 8, X  $2 < X < 18$
- $$\begin{array}{ll} 10 + X > 8 & X > \cancel{2} \\ 8 + X > 10 & X > 2 \\ 10 + 8 > X & 18 > X \end{array}$$
- 14) 10, 11, X  $1 < X < 21$
- $$\begin{array}{ll} 10 + 11 > X & 21 > X \\ 10 + X > 11 & X > 1 \\ 11 + X > 10 & X > \cancel{1} \end{array}$$
- 15) 8, 6, X  $2 < X < 14$
- $$\begin{array}{ll} 8 + 6 > X & 14 > X \\ 8 + X > 6 & X > \cancel{2} \\ 6 + X > 8 & X > 2 \end{array}$$

# Simplest Radical

Find the area of each triangle. Round your final answer to 2 places.

16)



$$P = 2\sqrt{2} + 5 + \sqrt{21} + 2$$

$$P = (7 + 2\sqrt{2} + \sqrt{21}) \text{ un.}$$

$$2^2 + 2^2 = x^2$$

$$8 = x^2$$

$$x = 2\sqrt{2}$$

$$2^2 + y^2 = 5^2$$

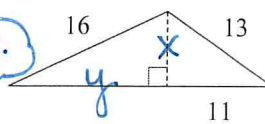
$$4 + y^2 = 25$$

$$y^2 = 21$$

$$y = \sqrt{21}$$

$$A = \frac{1}{2}(2 + \sqrt{21})(2)$$

$$A = \frac{1}{2}(4 + 2\sqrt{21}) = (2 + \sqrt{21}) \text{ un.}^2$$



$$11^2 + x^2 = 13^2$$

$$121 + x^2 = 169$$

$$x^2 = 48$$

$$x = 4\sqrt{3}$$

$$(4\sqrt{3})^2 + y^2 = 16^2$$

$$48 + y^2 = 256$$

$$y^2 = 208$$

$$y = \sqrt{208} = 4\sqrt{13}$$

$$P = 11 + 13 + 11 + 4\sqrt{3}$$

$$P = (40 + 4\sqrt{3}) \text{ un}$$

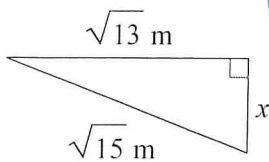
$$A = \frac{1}{2}(11 + 4\sqrt{3})(4\sqrt{3})$$

$$A = \frac{1}{2}(44\sqrt{3} + 16\sqrt{39})$$

$$A = (22\sqrt{3} + 8\sqrt{39}) \text{ un.}^2$$

Find the missing side of each triangle. Leave your answers in simplest radical form.

18)



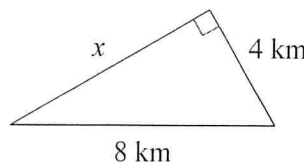
$$(\sqrt{13})^2 + x^2 = (\sqrt{15})^2$$

$$13 + x^2 = 15$$

$$x^2 = 2$$

$$x = \sqrt{2} \text{ m}$$

19)



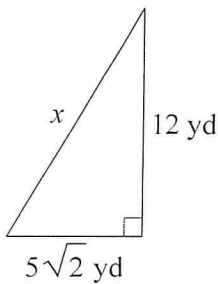
$$x^2 + 4^2 = 8^2$$

$$x^2 + 16 = 64$$

$$x^2 = 48$$

$$x = 4\sqrt{3} \text{ km}$$

20)



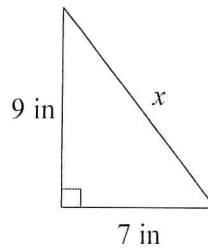
$$12^2 + (5\sqrt{2})^2 = x^2$$

$$144 + 50 = x^2$$

$$194 = x^2$$

$$x = \sqrt{194} \text{ yd}$$

21)



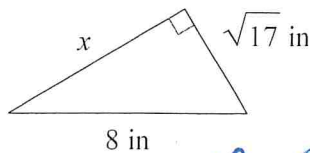
$$9^2 + 7^2 = x^2$$

$$81 + 49 = x^2$$

$$130 = x^2$$

$$x = \sqrt{130} \text{ in}$$

22)



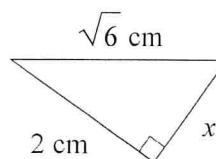
$$x^2 + (\sqrt{17})^2 = 8^2$$

$$x^2 + 17 = 64$$

$$x^2 = 47$$

$$x = \sqrt{47}$$

23)



$$2^2 + x^2 = (\sqrt{6})^2$$

$$4 + x^2 = 6$$

$$x^2 = 2$$

$$x = \sqrt{2} \text{ cm}$$

## Answers to

1) No

5) No

9) Obtuse

13)  $2 < x < 18$

17) 87.6

21)  $\sqrt{130}$  in

2) Yes

6) Yes

10)  $4 < x < 18$

14)  $1 < x < 21$

18)  $\sqrt{2}$  m

22)  $\sqrt{47}$  in

3) Yes

7) Right

11)  $0 < x < 24$

15)  $2 < x < 14$

19)  $4\sqrt{3}$  km

23)  $\sqrt{2}$  cm

4) No

8) Acute

12)  $3 < x < 15$

16) 6.6

20)  $\sqrt{194}$  yd

KEY

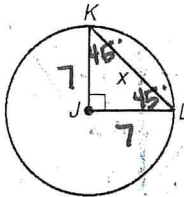
LESSON  
5-8

# Challenge

## Applying Properties of Special Right Triangles

Use the properties of special right triangles to solve each problem.  
Give your answers in simplest radical form.

1. The circumference of circle  $J$  is  $14\pi$ .  
What is the value of  $x$ ?



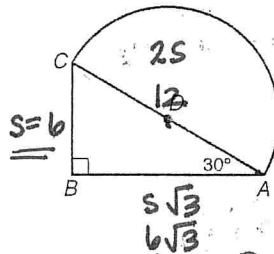
$$C = 2\pi r$$

$$14\pi = 2\pi r$$

$$7 = r$$

$$x = 7\sqrt{2}$$

2. The area of semicircle  $D$  is  $18\pi$ . What is the perimeter of  $\triangle ABC$ ?



$$A = \frac{1}{2}\pi r^2$$

$$18\pi = \frac{1}{2}\pi r^2$$

$$36 = r^2$$

$$r = 6$$

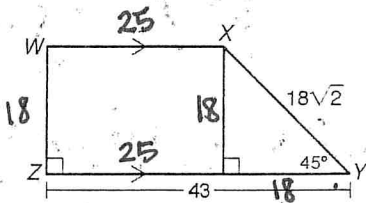
$$d = 12$$

$$s = 6$$

$$P = 12 + 6 + 6\sqrt{3}$$

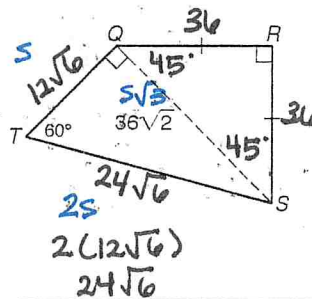
$$P = 18 + 6\sqrt{3}$$

3. Find the perimeter of quadrilateral  $WXYZ$ .



$$18 + 25 + 18\sqrt{2} + 43 = 86 + 18\sqrt{2}$$

4. Find the perimeter of quadrilateral  $QRST$ .



$$\frac{36\sqrt{2}}{\sqrt{3}} = \frac{s\sqrt{3}}{\sqrt{3}}$$

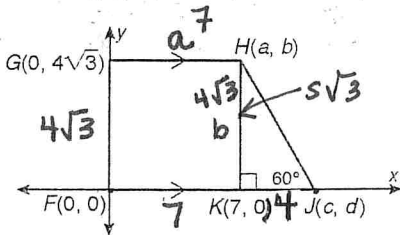
$$\frac{36\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{36\sqrt{6}}{3}$$

$$s = 12\sqrt{6}$$

$$P = 36 + 36 + 24\sqrt{6} + 12\sqrt{6}$$

$$P = 72 + 36\sqrt{6}$$

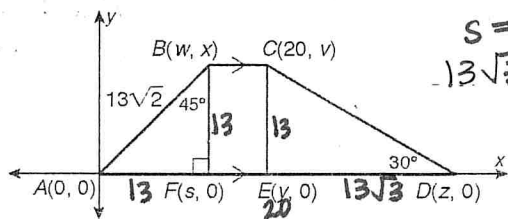
5. Find  $a$ ,  $b$ ,  $c$ , and  $d$ .



$$a = 7 \quad c = 11$$

$$b = 4\sqrt{3} \quad d = 0$$

6. Find  $w$ ,  $x$ ,  $y$ , and  $z$ .



$$s = 13$$

$$13\sqrt{3}$$

$$w = 13 \quad y = 20$$

$$x = 13 \quad z = 20 + 13\sqrt{3}$$

$$4\sqrt{3} = s\sqrt{3}$$

$$s = 4$$

$$c - 7 = 4$$

$$c = 11$$

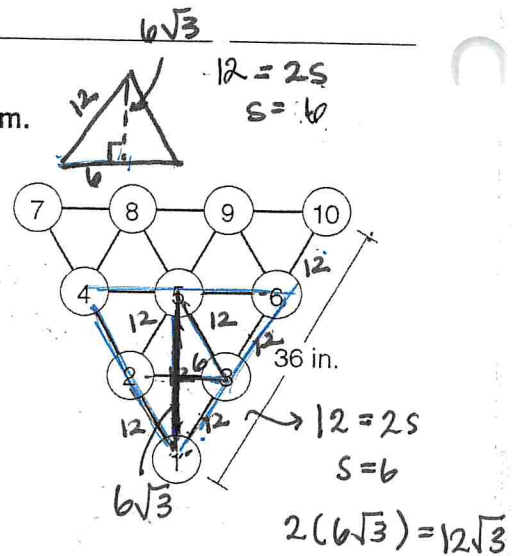
**LESSON**  
**5-8**

**Problem Solving**  
**Applying Special Right Triangles**

For Exercises 1–6, give your answers in simplest radical form.

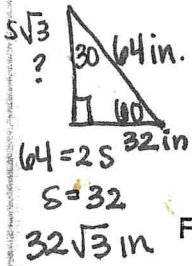
1. In bowling, the pins are arranged in a pattern based on equilateral triangles. What is the distance between pins 1 and 5?

$2(6\sqrt{3}) = 12\sqrt{3} \text{ in.}$

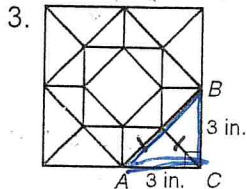


2. To secure an outdoor canopy, a 64-inch cord is extended from the top of a vertical pole to the ground. If the cord makes a  $60^\circ$  angle with the ground, how tall is the pole?

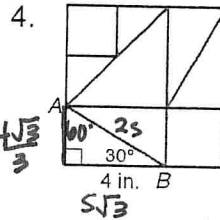
$32\sqrt{3} \text{ in.}$



Find the length of  $\overline{AB}$  in each quilt pattern.

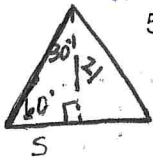


$\frac{3}{\sqrt{2}} = \frac{l\sqrt{2}}{\sqrt{2}}$   
 $l = \frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$   
 $\overline{AB} \Rightarrow 2\left(\frac{3\sqrt{2}}{2}\right) = 3\sqrt{2}$



$\frac{4}{\sqrt{3}} = \frac{s\sqrt{3}}{\sqrt{3}}$   
 $s = \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$   
 $\overline{AB} \Rightarrow 2\left(\frac{4\sqrt{3}}{3}\right) = \frac{8\sqrt{3}}{3}$

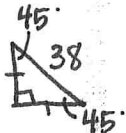
Choose the best answer.



5. An equilateral triangle has an altitude of 21 inches. What is the side length of the triangle?

$\frac{s\sqrt{3}}{2} = 21 \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{21\sqrt{3}}{3} = 7\sqrt{3}$   
side:  $2(7\sqrt{3}) = 14\sqrt{3}$

6. A shelf is an isosceles right triangle, and the longest side is 38 centimeters. What is the length of each of the other two sides?



$38 = l\sqrt{2}$   
 $l = \frac{38}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{38\sqrt{2}}{2} = 19\sqrt{2}$

Use the figure for Exercises 7 and 8.  $14\sqrt{3}$

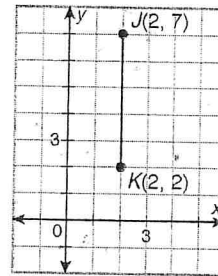
Assume  $\triangle JKL$  is in the first quadrant, with  $m\angle K = 90^\circ$ .

7. Suppose that  $\overline{JK}$  is a leg of  $\triangle JKL$ , a  $45^\circ$ - $45^\circ$ - $90^\circ$  triangle. What are possible coordinates of point L?

- A (6, 4.5)      C (6, 2)  
**B (7, 2)**      D (8, 7)

8. Suppose  $\triangle JKL$  is a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle and  $\overline{JK}$  is the side opposite the  $60^\circ$  angle. What are the approximate coordinates of point L?

- F (4.9, 2)**      H (8.7, 2)  
G (4.5, 2)      J (7.1, 2)



$\frac{5}{\sqrt{3}} = \frac{s\sqrt{3}}{\sqrt{3}}$

$s = \frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3} \approx 2.89$

$2 + 2.89 = 4.89$

Skip



P. 365, #2-17

2.  $\angle L, \angle K, \angle M$

3.  $\overline{EF}, \overline{DE}, \overline{DF}$

4.  $8.3 + 10.5 > 18.8 \times$   
 $8.3 + 18.8 > 10.5 \checkmark$   
 $10.5 + 18.8 > 8.3 \checkmark$  } NO!

5.  $4s, s+10, s^2, s=4$   
 $16, 14, 16$   
 $16 + 14 > 16 \checkmark$   
 $16 + 16 > 14 \checkmark$  } YES!

6.  $9 \text{ km}, 16 \text{ km}, x \text{ km}$   
 $9 + 16 > x$   
 $25 > x$   
 $x < 25$   
 $x + 9 > 16$   
 $x > 7$   
 $x + 16 > 9$   
 $x > -7$   
 $7 < x < 25$   
 km km

7.  $PR > SV$

8.  $m \angle KJL < m \angle MJL$

9.  $4x - 13 < 15$   
 $4x < 28$   
 $x < 7$   
 $4x - 13 > 0$   
 $4x > 13$   
 $x > 3.25$   
 $3.25 < x < 7$

10.  $5^2 + 9^2 = x^2$   
 $25 + 81 = x^2$   
 $106 = x^2$   
 $x = \sqrt{106}$

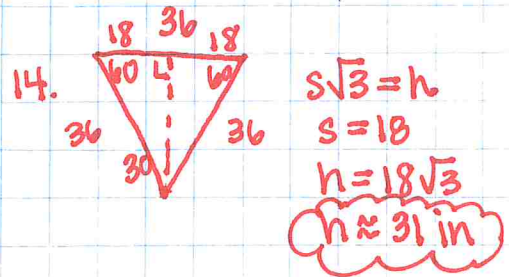
11.  $s\sqrt{3} = 5\sqrt{3} \rightarrow s = 5$   
 $x = 5$   
 $y = 2(5) = 10$

11.  $11^2 = 9^2 + x^2$   
 $121 = 81 + x^2$   
 $x^2 = 40$   
 $x = \sqrt{40}$   
 $x = \sqrt{4 \cdot 10}$   
 $x = 2\sqrt{10}$

12.  $10 + 12 > 16 \checkmark$   
 $10 + 16 > 12 \checkmark$  YES  
 $12 + 16 > 10 \checkmark$   
 $16^2 \quad 10^2 + 12^2$   
 $256 \quad 100 + 144$   
 $256 \quad 244$

$\therefore$  OBTUSE

13.  $80^2 + 50^2 = x^2$   
 $6400 + 2500 = x^2$   
 $8900 = x^2$   
 $x = 94.34$   
 $94 \text{ ft}, 4 \text{ in}$   
 $0.34 \cdot 12 = 4.08$



15.  $d = 8, x = 8\sqrt{2}$

16.  $\frac{d\sqrt{2}}{\sqrt{2}} = \frac{22}{\sqrt{2}}$   
 $d = \frac{22}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{22\sqrt{2}}{2} = 11\sqrt{2}$

#17 at the top ↑

17.  $x^2 + 2x + 1 = 0$   
 $(x+1)^2 = 0$   
 $x+1 = 0$   
 $x = -1$

18.  $x^2 + 4x + 4 = 0$   
 $(x+2)^2 = 0$   
 $x+2 = 0$   
 $x = -2$

19.  $x^2 + 6x + 9 = 0$   
 $(x+3)^2 = 0$   
 $x+3 = 0$   
 $x = -3$

20.  $x^2 + 8x + 16 = 0$   
 $(x+4)^2 = 0$   
 $x+4 = 0$   
 $x = -4$

21.  $x^2 + 10x + 25 = 0$   
 $(x+5)^2 = 0$   
 $x+5 = 0$   
 $x = -5$

22.  $x^2 + 12x + 36 = 0$   
 $(x+6)^2 = 0$   
 $x+6 = 0$   
 $x = -6$

23.  $x^2 + 14x + 49 = 0$   
 $(x+7)^2 = 0$   
 $x+7 = 0$   
 $x = -7$

24.  $x^2 - 2x + 1 = 0$   
 $(x-1)^2 = 0$   
 $x-1 = 0$   
 $x = 1$

25.  $x^2 - 4x + 4 = 0$   
 $(x-2)^2 = 0$   
 $x-2 = 0$   
 $x = 2$

26.  $x^2 - 6x + 9 = 0$   
 $(x-3)^2 = 0$   
 $x-3 = 0$   
 $x = 3$

27.  $x^2 - 8x + 16 = 0$   
 $(x-4)^2 = 0$   
 $x-4 = 0$   
 $x = 4$

28.  $x^2 - 10x + 25 = 0$   
 $(x-5)^2 = 0$   
 $x-5 = 0$   
 $x = 5$

29.  $x^2 - 12x + 36 = 0$   
 $(x-6)^2 = 0$   
 $x-6 = 0$   
 $x = 6$

30.  $x^2 - 14x + 49 = 0$   
 $(x-7)^2 = 0$   
 $x-7 = 0$   
 $x = 7$

p. 370, # 10-20

10.  $\angle E, \angle B, \angle H$

11.  $\overline{TY}, \overline{RY}, \overline{RT}$

12.  $114, 247, x$   
mi mi mi

$$114 + 247 > x$$

$$361 > x$$

$$x < 361$$

$$114 + x > 247$$

$$x > 133$$

$$247 + x > 114$$

$$x > -133$$

$133 < x < 361$   
mi mi

13.  $m < SPV < m < ZPV$

14.  $4x - 10 < 24$   
 $4x < 34$   
 $x < 8.5$

$4x - 10 > 0$   
 $4x > 10$   
 $x > 2.5$

$2.5 < x < 8.5$

15.  $21^2 + x^2 = 24^2$   
 $441 + x^2 = 576$   
 $x^2 = 135$   
 $x \approx 11.62$

21, 24, 11.62

not a Pythag. Triple

16. 18, 20, 27

$18 + 20 > 27 \checkmark$   
 $18 + 27 > 20 \checkmark$   
 $20 + 27 > 18 \checkmark$   
YES

$18^2 + 20^2 \quad \underline{\quad} \quad 27^2$   
 $324 + 400 \quad \underline{\quad} \quad 729$   
 $724 \quad \underline{\quad} \quad 729$

OBTUSE

17.  $6^2 + 8^2 = x^2$   
 $36 + 64 = x^2$   
 $100 = x^2$

$x = 10$   
 $102 \text{ ft}, 10 \text{ in}$

18.  $\frac{l\sqrt{2}}{\sqrt{2}} = \frac{20}{\sqrt{2}}$

$$l = \frac{20}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{20\sqrt{2}}{2}$$

$$l = 10\sqrt{2}$$

$x = 10\sqrt{2}$

19.  $32 = 2s$

$$s = 16$$

$$x = 16$$

$$y = 16\sqrt{3}$$

20.  $\frac{8}{\sqrt{3}} = \frac{s\sqrt{3}}{\sqrt{3}}$

$$s = \frac{8}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$$

$$x = \frac{8\sqrt{3}}{3}$$

$$y = 2 \left( \frac{8\sqrt{3}}{3} \right)$$

$$y = \frac{16\sqrt{3}}{3}$$

